ENERGY IN SIMPLE HARMONIC MOTION (SHM)

Unit–1 : Basic of Oscillations

Prepared by

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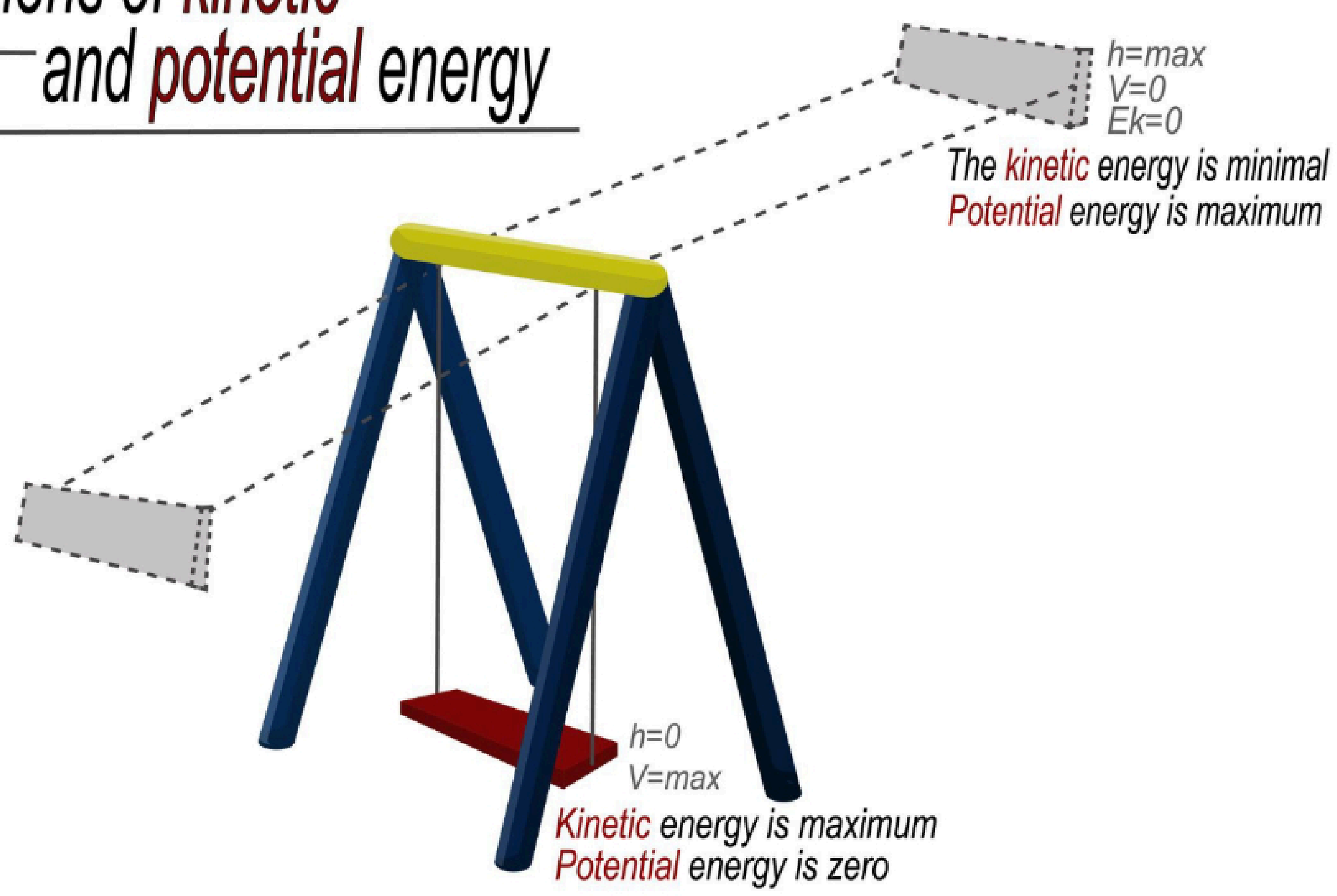
1. Introduction

Simple Harmonic Motion (SHM) is a periodic motion in which the restoring force acting on a particle is directly proportional to its displacement from the mean position and is always directed towards it.
In SHM, energy continuously transforms between **kinetic energy** and **potential energy**, while the **total mechanical energy remains constant** (in ideal conditions).

Transformations of kinetic and potential energy



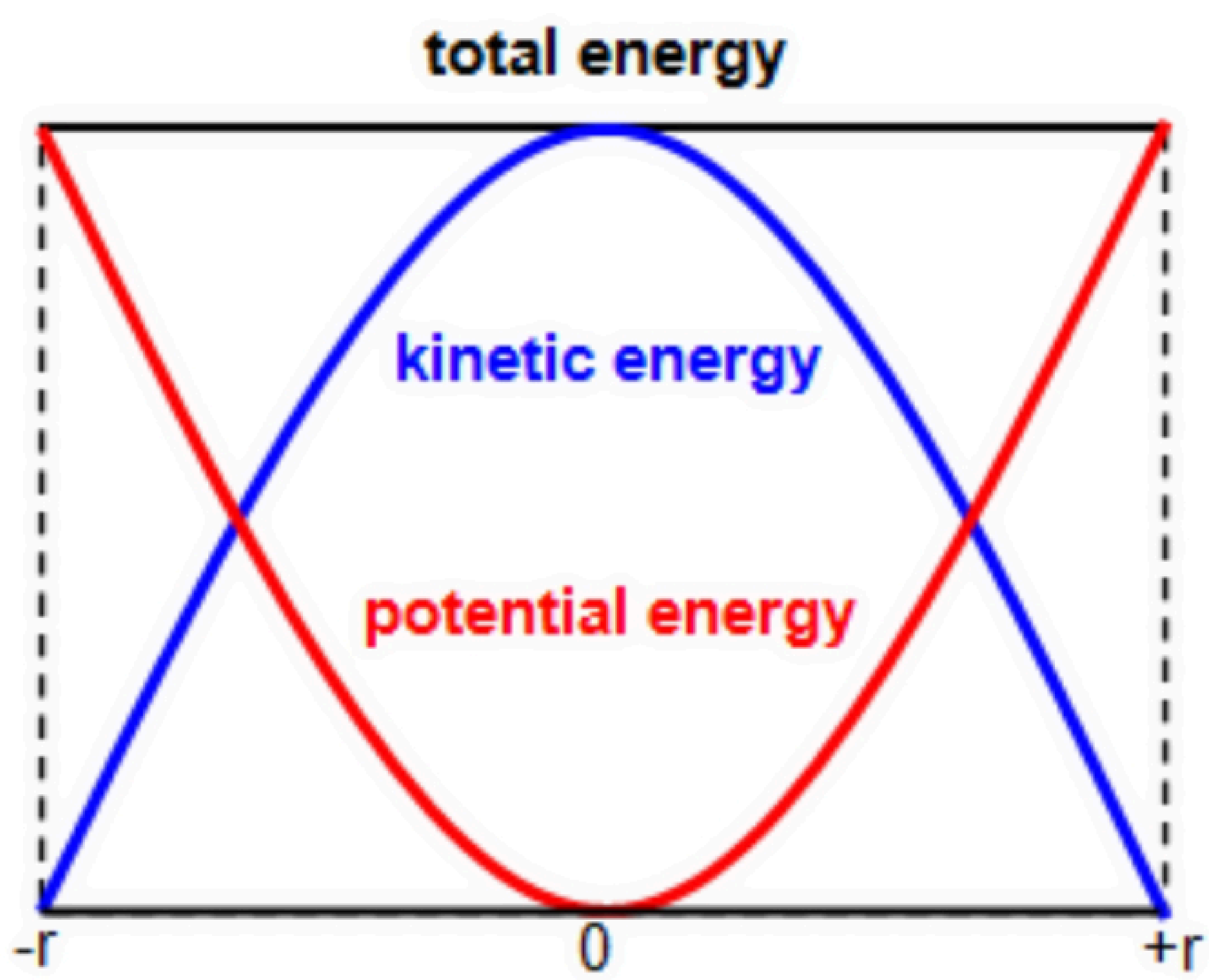
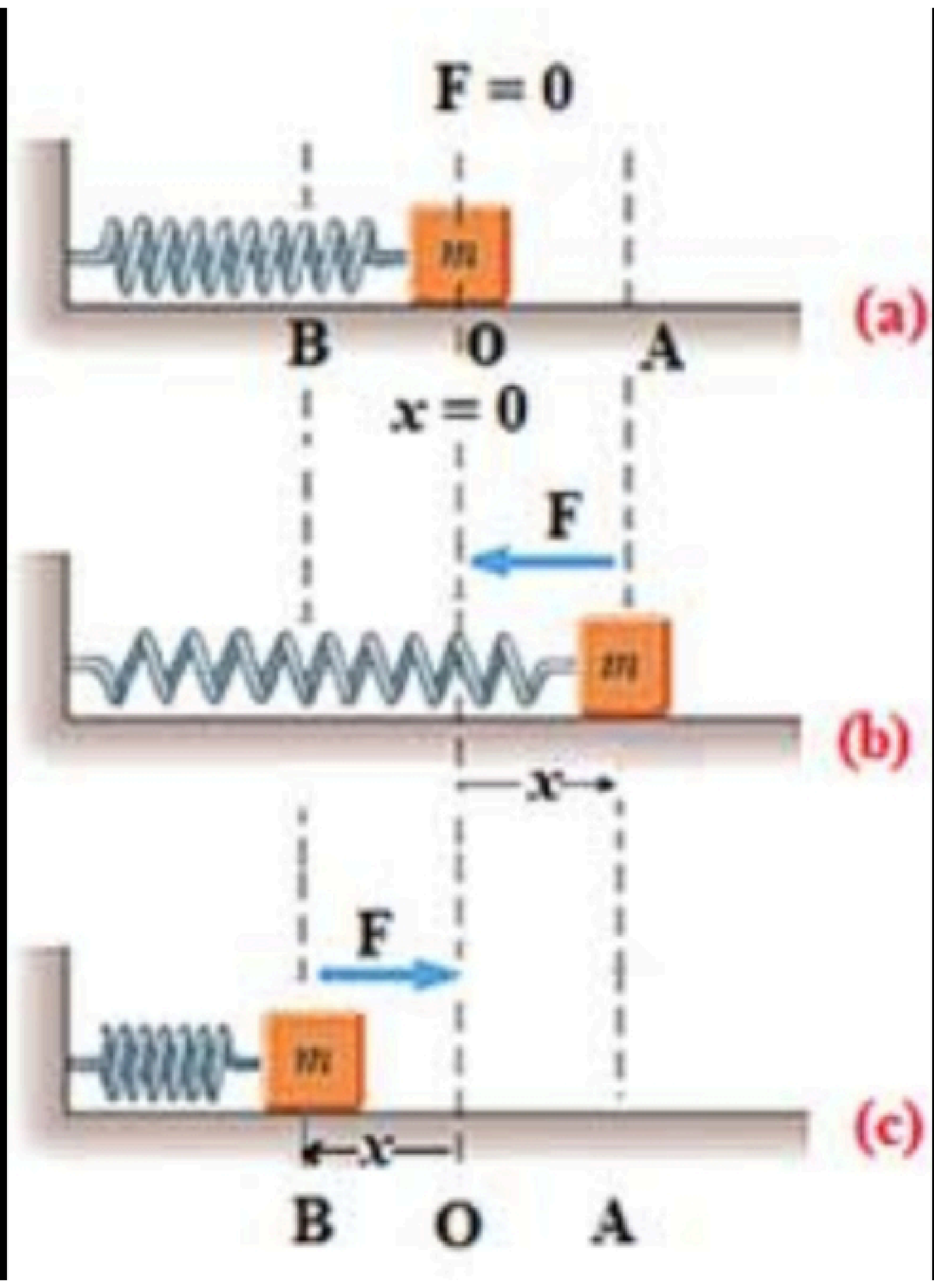


Figure 1



F = 0
(a)
B
O
A
x = 0
F
(b)
x
F
(c)
x
B
O
A

2. Expression for SHM

The displacement of a particle executing SHM is given by

$$x = A\sin(\omega t + \phi)$$

- A = amplitude
- ω = angular frequency
- ϕ = phase constant

Velocity,

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$$

3. Kinetic Energy in SHM

The kinetic energy (K.E.) of a particle of mass m moving with velocity v is

$$\text{K.E.} = \frac{1}{2}mv^2$$

Substituting the value of velocity:

$$\text{K.E.} = \frac{1}{2}m\omega^2(A^2 - x^2)$$

Important points

- K.E. is **maximum at the mean position** $(x = 0)$
- K.E. is **zero at the extreme positions** $(x = \pm A)$

4. Potential Energy in SHM

The restoring force in SHM is

$$F = -kx$$

Potential energy (P.E.) is given by

$$\text{P.E.} = \int F\,dx = \frac{1}{2}kx^2$$

Since $k = m\omega^2$,

$$\text{P.E.} = \frac{1}{2}m\omega^2x^2$$

Important points

- P.E. is **minimum (zero) at the mean position**
- P.E. is **maximum at extreme positions**

5. Total Mechanical Energy in SHM

Total energy (E) is the sum of kinetic and potential energies:

$$E = \text{K.E.} + \text{P.E.}$$

$$E = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2x^2$$

$$E = \frac{1}{2}m\omega^2A^2$$

Result

👉 **Total energy in SHM is constant and independent of displacement or time.**

6. Energy at Different Positions

Position	Kinetic Energy
Mean position ($x = 0$)	Maximum
Extreme position ($x = \pm A$)	Zero
Any intermediate position	Partial

7. Variation of Energy with Displacement

- K.E. varies as $(A^2 - x^2)$
- P.E. varies as x^2
- Total energy remains constant

The energy curves of K.E. and P.E. are parabolic, while total energy is a straight line parallel to the displacement axis.

8. Physical Significance

- SHM represents continuous conversion of energy
- No loss of energy occurs in ideal SHM
- Basis of oscillatory systems like springs, pendulums, molecular vibrations, etc.

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9. Conclusion

In Simple Harmonic Motion, although kinetic and potential energies change continuously, the **total mechanical energy remains conserved**. This principle plays a crucial role in understanding oscillatory systems in classical mechanics.